Negative Group Velocity of Lamb-Type Wave on Solid/Liquid/Solid Structure

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1. Introduction

Lamb waves1) propagating in an elastic plate have negative group velocities that the energy transport is in the opposite direction to the phase velocity in certain conditions. The negative group velocities appear near cut-off frequencies of certain propagation modes2). A range of their existence has already been got the result, which depends on Poisson’s ratio3). In other word, the negative group velocities of Lamb waves can not be changed, dynamically. Correspondingly, the negative group velocities of Lamb-type waves on glass/water/glass structure can be changed by changing the thickness of water layer, dynamically4). In this research, it was investigated that the existence of the negative group velocities of Lamb-type waves on solid/liquid/solid structure with respect to physical parameters of the structure. As a result, it was identified that the range of the existence depends not only Poisson’s ratio but also the density.

2. Lamb-type waves

We consider the thin water layer embedded between two identical elastic plates. This structure is shown in Fig. 1. When boundaries \( z = \pm d_W / 2 \) contact the water layer, Lamb waves partly leak longitudinal waves into the water layer with their propagations. And so, Lamb-type waves as a coupling mode of leaky Lamb waves propagate in the solid/liquid/solid structure. Dispersion relation5,6) of Lamb-type wave in the solid/liquid/solid structure is written as

\[
\tau = \frac{\rho_W}{\rho} k_{2W}^4 - k_{2W}^2, \quad (4)
\]

\[
k_{2W} = \sqrt{k_W^2 - k^2}. \quad (5)
\]

\( D_S \) and \( D_A \) are the dispersion relations of symmetrical and antisymmetrical modes of the Lamb waves in an elastic plate, respectively.

In the water layer, the parameter: water density, sound velocity in water, and water wave number, are \( \rho_W, c_W, \) and \( k_W = \omega / c_W \), respectively. Also, elastic plate density is \( \rho \). We define the modes whose deformation on the surface \( z = \pm (d_W / 2 + d) \) is symmetric with respect to the central plane as global symmetrical (GS) modes. Global antisymmetrical (GA) modes are also defined in the same manner except that the deformation is antisymmetric. Additionally, each mode can be divided into two modes with respect to two identical elastic plates. GS-S mode is the global symmetrical mode that consists of the two symmetrical modes of elastic plates.

\[
D_{GS}(\omega,k)D_{GA}(\omega,k) = 0, \quad (1)
\]

with:

\[
D_{GS}(\omega,k) = 2D_SD_A + \tau(D_A - D_S)\cot(k_{2W}d_W / 2), \quad (2)
\]

\[
D_{GA}(\omega,k) = 2D_SD_A - \tau(D_A - D_S)\tan(k_{2W}d_W / 2), \quad (3)
\]

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Fig. 1 Solid/Liquid/Solid structure that Lamb-type waves propagate used on numerical analysis

Fig. 2 Mode shapes of Lamb-type waves that are composed of Solid/Liquid/Solid structure
GS-A mode is the global symmetrical mode that consists of the two antisymmetrical modes of elastic plates. The GA-S mode and the GA-A mode are defined, similarly. The mode shapes of these are shown in Fig. 2. Here, the Lamb-type waves we consider are coupling mode composed of some modes.

3. Existence of negative group velocity

The wave number of each mode is obtained by solving eqs. (2) and (3), and the group velocities are obtained by solving equation \( \frac{d \varphi}{d \rho} \). Lamb wave has velocity dispersion that the phase velocity depends on the product \( f d \) (\( f \):frequency, \( d \): the thickness of the elastic plate). Obviously, Lamb-type wave has such characteristics, too. The negative group velocities appear near cut-off frequencies of certain propagation modes. The cut-off frequency means the point at which the phase velocity is divergent. The relation between the existence range of negative group velocity \( \Delta f \) and the cut-off frequency \( f_c \) is shown in Fig. 3. The phase velocity \( c_p \) and the group velocity \( c_g \) with respect to \( f d \) are shown. The negative group velocity exists where the gradient sign of the phase velocity turns over.

In this research, the Lamb-type waves of GS-S1 mode in the condition \( d/d \rho = 0.016 \) were considered about the existence of negative group velocity with respect to physical parameters. As a result, it was identified the existence depends not only Poisson’s ratio but also the density of the elastic plates in contrast to the case of Lamb waves on an elastic plate. The results are shown in Fig. 4. The top and middle figures show the \( \Delta f f_c \) as a existence range of negative group velocity respect to Poisson’s ratio \( \sigma \). The bottom figure shows the \( \max (-c_g) \) respect to Poisson’s ratio \( \sigma \). Here, in order to change Poisson’s ratio \( \sigma \), the transverse wave velocity is changed while the longitudinal sound velocity is fixed by 5000 m/s. In Lamb-type waves, it was seen that the existence range of negative group velocity changed widely depending on the density \( \rho \). The peaks of lines composed of same density \( \rho \) of \( \Delta f f_c \) and \( \max (-c_g) \) are sharp and move in the direction of high Poisson’s ratio \( \sigma \) as the density \( \rho \) is high, in most case. But, in the \( \Delta f f_c \), the peaks move in the direction of low Poisson’s ratio \( \sigma \) in case that \( \rho \) is larger than about \( 21 \times 10^3 \) kg/m^3.

4. Conclusion

In this research, we calculated the existence range of negative group velocity of Lamb-type waves in solid/liquid/solid structure depending on physical parameters. As a result, It is confirmed that the negative group velocities depend not only on Poisson’s ratio but also on the density of the elastic plates. We expect that this result can be applied to fabricating flat lenses of acoustical devices, etc.

References